

Branch-and-price for the multi-depot pickup and delivery problem with heterogeneous fleet and soft time windows

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Key words: branch-and-price, pickup and delivery, soft time windows

1 Introduction

The multi-depot pickup and delivery problem with heterogeneous fleet and soft time windows (MDPDPHSTW) requires to find a minimum cost routing for a fleet vehicles with different capacities and based at different depots, satisfying a given set of customers. A customer request is associated with two locations: a source where a certain demand must be picked up and a destination where this demand must be delivered. Each route must satisfy pairing constraints (pickup and delivery of a customer must both be visited in the same route) and precedence constraints (the delivery location must be visited after the corresponding pickup location). Further, each pickup and delivery location has a time window for the service that can be violated at the cost of a linear penalty. This problem has application in various scenarios, such as urban courier services, less-than-truckload transportation, door to door transportation services.

The problem has been widely studied in the version with hard time windows constraints (PDPTW). Several exact approaches based on branch-and-cut [1,8] and branch-and-price [3,10,7] have been proposed. For comprehensive reviews on routing problems involving pickup and delivery, the reader is referred to the works of Savelsberg and Sol [9], Cordeau et al. [2] and Parragh et al. [5]. The soft time windows case was address in an early work by Sexton and Choi [11]. They developed a heuristic algorithm based on Benders' decomposition.

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Recently, Liberatore et al. [4] and Qureshi et al. [6] proposed branch-and-price approaches for the vehicle routing problem with soft and semi-soft time windows.

In this work we propose a branch-and-price algorithm for the MDPDPHSTW which combines ideas proposed in [7] with bidirectional label extension and decremental state space relaxation, and uses a modified version of the algorithm developed by Liberatore et al. [4] to handle soft time windows.

2 Problem description

Given a set \mathcal{K} of vehicle types, a set \mathcal{H} of depots and a set \mathcal{N} of customer requests, the problem can be defined on a directed graph $G = (V, A)$, where V contains a vertex for each depot $h \in \mathcal{H}$ and for the pickup and delivery locations of each customer $i \in \mathcal{N}$. Non negative weights d_{ij} and t_{ij} are associated with each arc $(i, j) \in A$; they represent the transportation cost and the traveling time respectively. A service time s_j and a time window $[a_j, b_j]$ are associated to each vertex $j \in V$; if the service at location j starts inside its time window no penalty is incurred, otherwise a linear penalty, proportional to the anticipation or delay through non-negative coefficients α_j and β_j respectively, has to be paid. If we call T_j the starting time of service at location j , the penalty term $\pi(T_j)$ is defined as follows:

$$\pi(T_j) = \begin{cases} \alpha_j(a_j - T_j) & \text{if } T_j < a_j \\ 0 & \text{if } a_j \leq T_j \leq b_j \\ \beta_j(T_j - b_j) & \text{if } T_j > b_j. \end{cases}$$

Each vehicle type $k \in \mathcal{K}$ has given capacity w_k and fixed cost f_k . A limited number of vehicles is available: at most u_{hk} vehicles of type $k \in \mathcal{K}$ can be based at depot $h \in \mathcal{H}$.

The objective is to minimize the sum of vehicles fixed costs and routing costs (including penalties), satisfying the following conditions: (a) all customers is served, (b) each customer is visited by only one vehicle, (c) each route begins at a depot and ends at the same depot, (d) the capacity of the associated vehicle is not exceeded, (e) pickup and delivery of a customer are performed in the same route, (f) the pickup vertices are visited earlier than the corresponding delivery vertices, (g) the number of available vehicles of each type for each depot is not exceeded.

3 Formulation

We consider a set covering formulation of the MDPDPHSTW. We say that a route is feasible if it satisfies conditions (b), (c), (d), (e) and (f). Let Ω_{hk} be the set of all feasible routes using a vehicle of type $k \in \mathcal{K}$ from depot $h \in \mathcal{H}$. We associate a binary variable x_r with each feasible route, which takes value 1 if and only if route r is selected. Let a_{ir} be a binary coefficient with value 1 if and only if customer i is visited by route r . Let c_r be the cost of route r ; it is equal to the sum of the vehicle fixed cost f_k and the routing costs. With these definitions we obtain the following integer linear programming model:

$$\min \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}} \sum_{r \in \Omega_{hk}} c_r z_r \quad (1)$$

$$\text{s.t.} \quad \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}} \sum_{r \in \Omega_{hk}} a_{ir} z_r \geq 1 \quad \forall i \in \mathcal{N} \quad (2)$$

$$\sum_{r \in \Omega_{hk}} z_r \leq u_{hk} \quad \forall h \in \mathcal{H}, k \in \mathcal{K} \quad (3)$$

$$z_r \in \{0, 1\} \quad \forall h \in \mathcal{H}, k \in \mathcal{K}, r \in \Omega_{hk} \quad (4)$$

Constraints (2) are standard set covering constraints, modeling condition (a), while (3) impose limits on the maximum number of available vehicles of each type at each depot, modeling condition (g). The objective is to minimize the overall cost of the selected routes. In the remainder we indicate this formulation as Master Problem (MP).

4 Branch-and-price

We solve the linear relaxation of the MP to obtain a lower bound which is used in a tree search algorithm. The number of variables is exponential in the cardinality of the customer set \mathcal{N} , thus we use a column generation approach.

Given a depot h and a vehicle type k , the problem of finding the most negative reduced cost column encoding a route for vehicle k using depot h turns out to be a *Resource Constrained Elementary Shortest Path Problem* (RCESPP). We solve it by using a dynamic programming algorithm whose structure is similar to the one proposed in [7], enriched with bidirectional label propagation and decremental state space relaxation techniques. Further, we need to deal with soft time windows. For this purpose we modify the algorithm proposed by Liberatore et al. [4] for the VRPSTW. The key idea is to store the cost of a path as a convex piecewise linear function of the time and to perform partial dominance on time intervals. In principle it would be necessary to solve, at each iteration, an instance of RCESPP for every combination of depot and

vehicle type, but in practice it is possible to avoid multiple executions. Two branching rules are used, one on the number of vehicles and the other one on the arcs used.

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